Shear and flexural deflection equations for OSB floor decking under point load

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ABSTRACT

With the ready availability of most of the mechanical properties of structural-grade OSB, their use in the assessment of the adequacy of OSB floor decking by calculation will include the determination of the deflection. However, shear and flexural deflection equations for orthotropic floor panel in terms of the bending and shear properties are scarce. The finite element method has enabled a parametric study of the peak shear and flexural deflections of rectangular plate models of joisted OSB floor decking. The study has considered the effects of decking continuity, orthotropy in bending, planar and panel shear stiffnesses and Poisson's ratio. Shear and flexural deflection equations of acceptable accuracy have been fitted to the deflections obtained from the finite element method. Case studies have also shown good agreement between the deflections obtained from the equations and finite element method. These equations are recommended for preliminary and detail designs.

INTRODUCTION

Until as recently as the late 1990's, designers have had to rely almost exclusively on OSB manufacturers for guidance on the deflection, safe load and safe span of structural-grade OSB panels. However, for wider structural use, from wood-frame construction to stress skin panels, the need to make the mechanical properties readily available to designers has been realized, especially in North America. With the on-going publication of the short and long-term mechanical properties, there is interest in design methods based on these properties.

In this regard, the point-load deflection of wood-based floor decking in terms of the structural properties is almost always important. The Structural Board Association quotes an equation (SBA, 1995) from the Wood Handbook of USDA Forest Service for the deflection at the center of an orthotropic sheathing with four supported edges and with the major axis perpendicular to the support. However, the equation is expressed only in terms of one set of basic material property, the bending moduli of elasticity. There is question about the suitability of this formula for OSB sheathing, given that no direct correlation has been shown to exist between the planar shear moduli of rigidity and the bending moduli of elasticity (Griffiths and Wickens (1995), Karacabeyli et al (1996)). Strictly, shear and flexural deflection equations expressed in terms of the shear and flexural properties are desired.

This focus of this study is on the deflection of joisted OSB floor sheathing with full edge support. Geometric models of fully supported joisted floor decking range from simply supported to continuous rectangular plates. The flexural deflection of a simply supported rectangular orthotropic plate with a concentrated load can be determined from classical plate equation (Timoshenko and Woinowsky-Krieger (1959) and Szilard (1974). The shear deflection component of the total deflection is significant in wood-based sheet material decking, but solution for the shear deflection of a simply supported rectangular plate is not readily available. In addition, shear and flexural deflection solutions for continuous rectangular plate with concentrated load are also scarce (Kearley and Carruthers, 1991).

The objective of this paper is to determine by parametric study the maximum shear and flexural deflections of simply supported and continuous rectangular plate models of joisted OSB floor sheathing with full edge support under concentrated load. Equations are fitted to the deflections for direct use in preliminary and detail designs.

ANALYSIS

Properties

In the parametric study, the actual values of the properties or the ratios of the properties in the principal directions are important. Extensive characteristic values of the mechanical properties of structural-grade OSB reported by Griffiths and

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Wickens (1995), the Structural Board Association (SBA, 1995) and Karacabeyli et al (1996) reveal that for structural-grade OSB the ratio of the major to minor bending MOE, $E_x/E_y$, ranges from 1.9 to 3.0, whilst the ratio of the major to minor planar shear moduli of rigidity, $G_{xz}/G_{yz}$, varies roughly from 0.8 to 1.2. The major and minor panel shear moduli of rigidity are also nearly equal. These planar and panel shear properties are consistent with the conclusions made from shear tests by Shrestha (1999) that the shear properties in the major and minor axes are practically equal. According to Bares (1971) and Szilard (1974), for an ideal orthotropic material, the panel shear modulus of rigidity, $G_{xy}$, is approximately equal to $0.5\sqrt{(E_xE_y)/(1 + \sqrt{(\nu_x\nu_y)})}$.

Data on the Poisson's ratio of OSB is scarce. For wood-based panels, the major Poisson's ratio in bending, $\nu_x$, appears to lie between 0.1 and 0.4 (Bodig and Jayne (1993), Hearmon and Adams (1952)). The major Poisson's ratio, $\nu_x$, is the surface strain in the minor axis per unit strain in the major axis when pure bending moment is applied in the major axis. Theoretically, the ratio of the Poisson's ratios, $\nu_x/\nu_y = E_x/E_y$, where $E_x$ and $E_y$ are major and minor moduli of elasticity in bending and $\nu_y$ is the minor Poisson's ratio in bending.

Structure and analysis
A joisted OSB panel floor comprises an OSB panel sub floor on a linear grid of joists. A secondary non-engineered set of supporting members, from noggings to blockings ensures robustness of the decking. It is assumed that the edge-joints between adjacent tongued-and-grooved panels are glued and/or supported by noggings or joists, while square-edged boards have all edges continuously supported on joists or noggings. Floors with unsupported panel edges are not covered in this study. The long edges or face grain direction or major material axis in the plane of the OSB decking is normally parallel to the span. As the joints along the shorter edges of the panels are staggered, single and continuous span panels occur near the perimeter of the floor. For nailed and screwed connection of the panel to the joists, the assumption of free rotation of the panels about the joists is made in routine design (Figure 1).

The thickness of OSB structural panels in joisted floor decking varies from 12 to 30 mm. Typical spans are 300, 400, 600 or 800 mm measured center-to-center of the joists. As the underlayments and/or finishes are not relied on to provide additional structural resistance to the applied load, the practical ratio of span to thickness ranges roughly from 10 to 40.

The floor decking reduces to a one-way spanning long rectangular panel. A rectangular plate with a ratio of width to span of 4.0 is assumed to model the surface area of the decking. The following assumptions are also made in the analysis: 1. The panels are midplane symmetric, linear elastic and are of identical type, grade and nominal thickness. 2. The floor decking is effectively restrained laterally. 3. The in-service load-deflection relationship for the decking is linear.

In the linear analysis a load of 1 kN is used. The side dimension of the area over which the concentrated load acts typically varies from 25 and 50 mm (Kearley and Carruthers, 1991). For such relatively small loaded areas, the deflection of the decking is almost independent of the magnitude of the loaded area. So, a point load was used throughout the analysis.

Thin and thick plate finite elements of the LUSAS Finite Element Analysis System were used to determine the deflections. The thick plate element is an eight-noded isoparametric plate element formulated using Mindlin plate theory. This element
accounts for the transverse shear effects associated with thick plates. It measures both the shear and flexural deflections. Assuming the rectangular Cartesian (x, y, z) axes are coincident with the principal property axes of the decking, the bending stiffness submatrix, \([D_{\text{bending}}]\) and shear stiffness submatrix, \([D_{\text{shear}}]\), are as follows

\[
[D_{\text{bending}}] = \frac{t^3}{12} \begin{bmatrix}
\frac{E_x}{(1-\nu_x\nu_y)} & -\nu_x E_y & 0 \\
-\nu_x E_y & \frac{E_y}{(1-\nu_x\nu_y)} & 0 \\
0 & 0 & G_{xy}
\end{bmatrix}
\]

where \(\nu_y = \nu_x E_y/E_x\) to maintain symmetry. As OSB is practically isotropic in planar shear, \(G_{xz} = G_{yz} = G_z\). The thin plate element is a four-noded quadrilateral isoparametric plate element produced by constraining the shear strains to zero at discrete points within the thick plate element. It measures only the flexural deflection.

A mesh size of one-tenth of the span was adopted in the analysis. For the purpose of analysis, the span and width of the model were taken as 1 m and 4 m respectively. Theoretically, the peak deflection occurs when the load is near the center of area between adjacent joists. In this analysis the critical load position is assumed to be the center of the area between adjacent joists.

The thin plate element was used for the analysis for peak flexural deflection. For the purpose of analysis, the major bending modulus of elasticity, \(E_x\), was assumed to be 1500 N/mm². The deflection under the load was determined for ratios of bending stiffnesses, \(E_x/E_y\), of 1.0, 2.0, 3.0 and 4.0 and in each case for Poisson’s ratio, \(\nu_x\), of 0.1, 0.2, 0.3 and 0.4. In order to study the effect of \(E_x/E_y\) at a fixed value of Poisson’s ratio, \(E_x\) was kept constant and equal to 1500 N/mm², whilst the value of \(E_y\) was taken as the variable. The value (theoretical) of \(G_{xy}\) was estimated from \(E_x, E_y, \nu_x\) and \(\nu_y\). The experimental and theoretical values differ in practice. A difference of about 20 percent relative to the theoretical value has been an OSB panel by Thomas (1996). For the results of analysis to be of practical use, the effect of deviation of the experimentally determined value of \(G_{xy}\) from the theoretical value, on the deflection, was assessed. The deflections due to values of \(G_{xy}\) of 0.7 and 1.3 times the theoretical value of \(G_{xy}\) were determined.

The thick plate deflection is the sum of the shear and flexural deflections. The shear deflection was calculated on the basis that for the same floor panel decking parameters, using thick and thin plate analyses, the difference between the deflection outputs is the shear deflection. The shear deflection of each model was assessed for span to thickness ratio, \(L/t\), of 40, 20 and 10 and \(G_x\) of \(E_x/16\), \(E_x/32\), \(E_x/64\) or \(E_x/80\). As there is no known closed form solution against which to compare the results, the shear deflections of typical single-span and continuous plate models were also calculated from thick plate analysis by assuming that the planar shear moduli of rigidity, \(G_x\), is fictitiously very large (1000\(E_x\)) in one case and real (\(E_x/16\), \(E_x/32\), \(E_x/64\) or \(E_x/80\)) in the other. The difference between the two deflections is the shear deflection. Analysis showed that the shear deflections were equal to those obtained from the use of thin and thick plate finite element analyses.

**RESULTS**

**Flexural deflection**

The suitability of the finite element model was based on comparison of the classical and finite element deflections of typical simply supported orthotropic plates. Compared to the classical solutions in Timoshenko and Woinowsky-Krieger (1959) and Szilard (1974), the errors in the maximum flexural deflections obtained from finite element analysis were of the order of one percent.

The variation of deflection with orthotropy in bending stiffness, \(E_x/E_y\), and major Poisson’s ratio (M.P.R), \(\nu_x\), is shown in Figure 2. The deflections have been normalized relative to the deflection of the isotropic model with a Poisson’s ratio of 0.1. The normalized profiles for the continuous model were found to be identical to those obtained for the single-span model. The theoretical value of \(G_{xy}\) has been assumed in the analysis. Deflections of the single-span and continuous plates for values of...
Gxy of 0.7 and 1.3 times the theoretical value of Gxy were assessed. Compared to the deflection when Gxy is equal to the theoretical value, the percentage changes in the flexural deflections are of the order of ±6%.

Shear deflection
The variation of the maximum shear deflection of the single-span plate is illustrated in Figure 3. For the ranges of Gz and L/t, the difference between the maximum shear deflections of the single-span and continuous plates expressed as a percentage of the single-span plate deflection is about 4%. The shear deflection is 0.768 mm for the simply supported single-span plate model with a planar shear stiffness, Gz, of Exy/64 (23.43 N/mm²) and a ratio of span to thickness of 20.

DISCUSSION

Flexural deflection
According to Figure 2, the flexural deflection is strongly dependent on the ratio of the bending moduli of elasticity, Ex/Ey, but moderately dependent on Poisson's ratio, νx. Compared to an isotropic plate of MOE, Ex, decrease of Ey such that Ex/Ey increases from 1.0 to 3.0 increases the flexural deflection by between 30 and 40 percent. As expected, the deflection is inversely proportional to the Poisson's ratio. Its significance diminishes with increase in the degree of orthotropy in bending stiffness.
The datum isotropic flexural deflections of the simply supported and continuous plates and the normalized deflection curves enable the derivation of deflection formulae for use in design. From the finite element analysis, the maximum flexural deflection of the simply supported single-span isotropic decking reduces to \( PL^2/59.1D \), where \( P \) is the applied load, \( L \) is the span and \( D (= E_t t^3/12(1-\nu^2)) \) is the plate flexural stiffness. This result compares with \( PL^2/59.6D \) given in Szilard (1974) and Timoshenko and Woinowsky-Krieger (1959). The maximum flexural deflection of the continuous model reduces to \( PL^2/74.6D \).

For design purposes, the equations to be developed should be simple but sufficiently accurate. The effect of \( E_x/E_y \) (i.e. for constant \( \nu_x \)) on the flexural deflection, \( w_f \), is fitted to the family of curves \( y = a + b\sqrt{x} \), where \( y \) is \( w_f \) and \( x \) is \( E_x/E_y \). \( a \) and \( b \) are coefficients to be determined. In the case of the effect of \( \nu_x \), the nature of the mainly non-linear spacing of the points on the curves when \( E_x/E_y \) is constant, is characteristic of the effect of Poisson's ratio in isotropic plates and suggests a quadratic polynomial (i.e. \( \propto \nu_x^2 \)). Minimization of the maximum difference between the analytical and fitted curves as the criterion for the determination of \( a \) and \( b \) yields the equation

\[
 w_f = \frac{PL^2}{cD_x} \left( 0.54 + 0.46 \left( \frac{E_x}{E_y} \right) (1 - 0.6 \nu_x^2) \right)
\]

(2)

where \( D_x = E_t t^3/12 \) and \( c \) is a constant equal to 59.1 and 74.6 for the single-span and continuous plates respectively. A plot of Equation 2 for Poisson's ratio, \( \nu_x \), of 0.1, 0.2, 0.3 and 0.4 is also shown in Figure 2. As the Poisson's ratios of OSB are scarce, a tentative median value of 0.2 or a conservative near minimum value of 0.1 can be assumed for \( \nu_x \) in design. The computed flexural deflection will be underestimated by 5 to 10% if the actual Poisson's ratio, \( \nu_x \), is of the order of 0.3 and 0.4. It is noted that the theoretical value of \( G_y \), which is a function of \( E_x, E_y, \nu_x \) and \( \nu_y \), has been used in the analysis and is indirectly reflected in Equation 2. Also, when the experimental \( G_{xy} \) is between 0.7 and 1.3 times the theoretical value, the deflection based on the theoretical value of \( G_{xy} \) is applicable with minimal error. Equation 2 suggests that the flexural deflection of the continuous decking is approximately 0.8 times the flexural deflection of the single-span decking.

Shear deflection

The shear deflections, \( w_s \), of the models, which are roughly isotropic in planar shear, have been found to be practically linearly dependent on \( PL/Gzt \) (Figure 3). This suggests an equation similar in form to the shear deflection of a simply supported isotropic beam under center-point load, which is given by \( 1.2P/(4Gzt) \), where \( G_z \), \( L \) and \( t \) are the same as the planar shear stiffness, span and thickness of the plate model respectively and \( b \) is the equivalent beam width. \( b \) is determined by equating the beam and plate shear deflections. By equating the slope of the plate shear deflection versus \( PL/Gzt \) to the slope of beam shear deflection versus \( PL/Gzt \), the equivalent beam width \( b \) is found to be 0.326 times the span. Substituting \( b = 0.326L \) in the equation for the beam deflection, the shear deflection of the single-span plate model reduces to

\[
 w_s = \frac{1.2PL}{4G_z t (0.326L)} = \frac{0.92P}{G_z t}
\]

(3)

Equation 3 is also applicable to the continuous model because the shear deflections of single-span and continuous models of the same span and material are approximately equal. ASCE (1975) confirms the approximate equality of the shear deflection of a simply supported panel with those for other support conditions.

Accuracy of equations

The maximum error in the flexural deflection equation is about \( \pm 9\% \). The maximum error is incurred close to the boundaries of the material parameters, that is, near \( E_x/E_y = 1.0 \) and 4.0 and \( \nu_x = 0.1 \) and 0.4. The ratio of bending stiffness, \( E_x/E_y \), for OSB typically lies between 2.0 and 3.0 and the maximum error that is likely to be incurred is about 6%.

From a comparison of the shear deflection equation and the shear deflection from finite element analysis, the error is \( \pm 2\% \). Case studies assessed the flexural and shear deflections of single-span and continuous OSB decking and obtained a maximum error of \( \pm 6\% \). In the general case, the values of \( G_{xy} \) and \( G_{yz} \) are different and have to be accommodated in designing. The maximum shear deflections of the single-span and continuous orthotropic models were found to be linearly dependent on the ratio \( G_{xy}/G_{yz} \) and fit the equation

\[
 w_s = \frac{0.92P}{G_z t} \left( 0.63 + 0.37 \frac{G_{xy}}{G_{yz}} \right)
\]

(4)
The maximum difference between the shear deflections of the single-span and continuous orthotropic models relative to the shear deflection of the single-span model was by coincidence of the order of 4%.

The values of $G_{xz}$ and $G_{yz}$ are required for effective use of Equations 3 and 4. However, relatively few test results are available. Published values of $G_{xz}$ and $G_{yz}$ are typically between 60 and 210 N/mm$^2$ (Griffiths and Wickens (1995), Karacabeyli et al (1996), Shrestha (1999)). In structural design, a useful gauge for the shear effect on deflection is the ratio of bending modulus of elasticity to planar shear modulus of rigidity, $E/G_z$. For structural-grade OSB, the stiffness ratio, $E_z/G_{xz}$, appears to vary from 50 to 75 whilst the ratio $E_y/G_{yz}$ seems to vary from 15 to 35. By comparison, for plywood, the ratio, $E/G_y$, is about 16. The relatively low planar shear stiffnesses could be a weak link in the structural application of OSB panels. Theoretically, the percentage of the total deflection that is due to planar shear is dependent on the ratio of span to thickness, the stiffness ratios, $E_x/G_{xz}$ and $E_y/G_{yz}$ and Poisson's ratio, $\nu_x$.

A typical calculation determines the deflection of an 18-mm thick OSB floor decking of 400-mm span in a 20°C and 80%RH environment. The imposed concentrated load is 2 kN and its cumulative duration is seven days. It is assumed that the edges of the boards are fully supported on joists and noggings. The decking is modeled as simply supported. The characteristic (mean) values of the 18-mm thick OSB are the following: bending stiffnesses, $E_x = 7000$, $E_y = 2500$; panel shear stiffnesses, $G_{xy} = G_{yx} = 1140$; planar shear stiffnesses, $G_{xz} = G_{yz} = 150$ and major Poisson's ratio, $\nu_x = 0.2$. Assuming the creep factor for load duration is roughly 1.0 and using Equation 2 the flexural deflection is 2.03 mm. Using Equation 3, the shear deflection is 0.68 mm. In this case, the shear deflection is 25% of the total deflection. Thus, unlike the USDA Forest Service equation (SBA, 1995), these equations for the shear and flexural components of the total deflection are expressed in rational formats for a calculation-based method of assessment of the deflection of joisted OSB floor decking.

**CONCLUSIONS**

The finite element method has enabled the determination of the effects of the material parameters, $E_x/E_y$, $G_{xz}/G_{yz}$, $\nu_x$ and $G_{xy}$, and panel continuity on the maximum flexural and shear deflections of rectangular plate models of OSB floor panel decking. The level of orthotropy in bending stiffness significantly influences the flexural deflection. Poisson's ratio moderately influences the flexural deflection. Although the theoretical value of $G_{xy}$ has been used in the determination of the flexural deflection, a difference of ±30% between the experimental and theoretical values of $G_{xy}$ produces only a ±6% change in the flexural deflection based on the theoretical value of $G_{xy}$. The flexural deflection of the continuous decking is about 0.8 times that of the single-span decking of the same material and span. The maximum shear deflections of the single-span and continuous orthotropic plate models of the same span and material properties are practically equal. As a result, the total deflection of the single-span decking is deemed to be roughly equal to the total deflection of the continuous decking.

The effective use of the fitted shear and flexural deflection equations depends on the availability of the elastic properties of OSB. Although the values of Poisson's ratios of OSB are scarce, the more important planar shear, panel shear and bending stiffnesses are continually being documented or are readily available.

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