Determination of transverse bending stiffness of nail-laminated timber elements

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ABSTRACT

There is a lack of research concerning the load bearing behaviour of nail laminated timber elements. The transverse bending stiffness is one significant value which is necessary in different cases. The known methods for determining the transverse bending stiffness of nail laminated timber elements do not reflect the real conditions in a satisfying way.

The objects of this work are an experimental and theoretical determination of the transverse bending stiffness. A new analytical approach for the calculation of the transverse bending stiffness is derived. The calculated values are compared to test results.

In a parameter study the values of bending stiffness are verified and the influence of the main parameters is shown.

COMPUTATIONAL MODEL

In the new approach - in accordance to the well-known model of a reinforced concrete beam at cracked state - the occurring state of strain is derived by the equilibrium of the compressive force in the wood and the tensile force of the nails. The knowledge of the neutral axis location and the height of the compression zone respectively, enables the determination of an equivalent bending stiffness. Thereby only the effective parts of the cross section are taken into consideration. These are the stiffness of the wood in the compression zone and the axial tensile nail compliance in the tension zone.

![Figure 1: Load-withdrawal displacement relationship of an annularly threaded nail 3.1x82mm](image)

The value of the bending stiffness is mainly influenced by the defined withdrawal stiffness of the applied nails. This

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characteristic value cannot be accessed directly for a special nail. It has to be deduced by the load-withdrawal relationship of a nail, which may be determined e.g. by withdrawal tests in accordance to the German code DIN 1052-2. Figure 1 shows the load-displacement curve of a test series for an annularly threaded nail 3.1x82 mm.

For the calculation model the withdrawal stiffness is assumed to be constant. It is defined by the quotient of the maximum withdrawal force and the respective displacement. (Figure 1)

\[
C_m = \frac{F_u}{s_u}
\]

(1)

\( C_m \) average withdrawal stiffness of a nail in N/mm
\( F_u \) ultimate withdrawal resistance in N
\( s_u \) respective withdrawal-displacement in mm

In a good approximation the joints between the boards can be neglected at the compression side. So the modulus of elasticity perpendicular to grain \( E_{\perp} \) of the boards is applied for the calculation of the material behaviour similar to a homogenous structure.

At the tension side, the compound of timber and nails determines the performance of the cross-section. The real load bearing behaviour can hardly be estimated and is approached by an elastic material-law. The appropriate modulus of elasticity is evaluated in a way, which reflects the load-deformation behaviour of a basic element (board + nail).

Equilibrium of forces, the compression force of the wood and the tensile force of the nail connection leads to the following formula concerning the height of the compression zone:

\[
h_D = -2 \cdot K_1 + \sqrt{4 \cdot K_1^2 + 2 \cdot K_1 \cdot (h_{N1} + h_{N2})}
\]

(2)

\[
K_1 = \frac{n \cdot t_B \cdot C_m}{E_{\perp}}
\]

(3)

\( h_D \) Compression zone height in mm
\( h_{N1} \) Distance of the lower nail row from the top edge of the nail-laminated timber element in mm
\( h_{N2} \) Distance of the upper nail row from the top edge of the nail-laminated timber element in mm
\( n \) Number of nails per meter and nail row, which satisfy the minimum driving depth according to DIN 1052 Part 2
\( t_B \) Thickness of single laminated member in mm
\( E_{\perp} \) Modulus of elasticity perpendicular to grain according to DIN 1052 Part 1 in N/mm²

Figure 2   Geometry of cross-section, strain distribution, internal forces.
The transverse bending stiffness relative to the neutral axis can be specified in dependence of the compression zone height:

\[
(\mathcal{E}I)_b = n \cdot E_s \cdot \frac{\pi}{32} d_n^3 + n \cdot C_m \cdot t_B \cdot \left( a_{N1}^2 + a_{N2}^2 \right) + E_s \cdot \frac{b \cdot h_D^3}{3} \tag{4}
\]

\[
a_{Ni} = h_{Ni} - h_D \tag{5}
\]

- \((\mathcal{E}I)_b\) Equivalent bending stiffness transverse to the board direction Nmm²/m
- \(E_s\) Modulus of elasticity of steel in N/mm²
- \(d_n\) Nail diameter in mm
- \(a_{N1}\) Distance of lower nail row to neutral axis in mm
- \(a_{N2}\) Distance of upper nail row to neutral axis in mm
- \(b\) 1000 mm width of the reference section

**BENDING TESTS**

To experimentally verify the derived model bending tests were carried out using beam-shaped nail-laminated timber elements. The test set-up was chosen as four-point loading arrangement as shown in figure 2, resulting in a constant moment span of approximately six-times element thickness. The system was loaded by two symmetrically arranged uniform loads at cantilever ends. This test set-up guarantees pure bending within the supports, so deformations will also be caused by pure bending. This is quite important for an exact determination of bending stiffness, because influence of shear deformation is not negligible considering the compliance of joints.

Preliminary experiments were carried out to obtain the withdrawal stiffness of the applied annularly threaded nails and the elastic modulus in compression perpendicular to grain of the chosen softwood, graded European S10. Both parameters are required for determination of bending stiffness.

![Figure 2](image)

In the performed test series dimensions and nail distances have been varied. The deflection \(w\) of the specimen was registered at mid-field. Figure 3 shows, that the bending stiffness derived by the mechanical model underestimates the real bending stiffness with the difference averaging about 40\%.
A possible explanation for the available larger bending stiffness may be represented by an increased nail-withdrawal stiffness in comparison to preliminary tests. This may be caused i.e. by resulting curvature of the loaded specimen and/or by unmethodically sloppy nailing. For clarification purposes further research will be undertaken.

**PARAMETER STUDY**

In this investigation the withdrawal test data of the annularly threaded nail RNa 31x65 is applied. The results of a test series according to DIN 1052-2 Appendix A are listed in table 1.

<table>
<thead>
<tr>
<th>Ultimate withdrawal resistance $F_u$</th>
<th>Withdrawal displacement for $F_u$ $s_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean value 1067,3 N 2,5 mm</td>
<td></td>
</tr>
<tr>
<td>Standard deviation 109 N 0,4 mm</td>
<td></td>
</tr>
</tbody>
</table>

As average value of withdrawal stiffness $C_m = F_u/s_u = 426,9$ N/mm is obtained. Here the withdrawal stiffness is kept constant. Its influence to the bending stiffness can be included by modifying the number of nails. The following objects should be examined by the parameter study:

- Influence of the number of nails respective the withdrawal stiffness of a nail to the bending stiffness
- Ratio of different parts of bending stiffness
- Influence of the board thickness to the bending stiffness.

The values of fixed parameters and ranges of varied parameters are specified in table 2.
Table 2  Parameter values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of elasticity of boards (NH S10)</td>
<td>$E_{\perp}$ 300 N/mm²</td>
</tr>
<tr>
<td>Modulus of elasticity of nail-wire</td>
<td>$E_{NA}$ 210000 N/mm²</td>
</tr>
<tr>
<td>Withdrawal stiffness of nail</td>
<td>$C_{m}$ 426.9 N/mm</td>
</tr>
<tr>
<td>Distance of the lower nail row to the bottom edge</td>
<td>$(h-h_{N1})$ 25 mm</td>
</tr>
<tr>
<td>Distance of the upper nail row to the top edge</td>
<td>$h_{N2}$ 25 mm</td>
</tr>
<tr>
<td>Board height</td>
<td>$h$ 60,...240 mm</td>
</tr>
<tr>
<td>Nail distance</td>
<td>$e_{NA}$ 40,...400 mm</td>
</tr>
<tr>
<td>Board thickness</td>
<td>$t_B$ 24,...80 mm</td>
</tr>
</tbody>
</table>

In the following diagrams, the results of the parameter study are presented graphically.

Figure 4 shows a nonlinear relationship between transverse bending stiffness and board height as well as between transverse bending stiffness and nail distance and effective withdrawal stiffness of nail connection respectively.

In Figure 5 the different parts of transverse bending stiffness and the bending stiffness in span direction for a nail-laminated timber element of 240 mm height are presented.

The part of transverse bending stiffness concerning the lower nail row is more than 90 %, so the parts of the wood and the upper nail row can be neglected in a good approximation. The real value of the equivalent modulus of elasticity for the compression zone, due to the joints between the boards is of minor importance.
The relationship of bending stiffness and board thickness is presented in Figure 6. There is nearly a linear correlation between board thickness and bending stiffness by the developed approach. Thereby it is assumed, that the withdrawal stiffness of the nails remains constant for the considered value of board thickness. In a comparative test the according properties of the connection have to be ensured.

\[(EI)_b \text{ [Nm}^2/\text{m]}\]

![Figure 6 Bending stiffness depending on board thickness (nail distance = 200 mm).](image)

**CONCLUSION**

The developed computational model enables the calculation of an analytically detectable value for the transverse bending stiffness of nail-laminated timber elements.

In accordance to the well-known model of reinforced concrete beams at cracked state, the occurring state of strain is derived by the equilibrium of the resulting compressive force in the wood and the tensile axial force of the nails. With the knowledge of neutral axis location and compression zone height respectively, the bending stiffness can be determined.
The value of transverse bending stiffness depends highly on the withdrawal stiffness of the nails. This characteristic value has to be determined by withdrawal nail tests.

By using the presented analytical approach the real transverse bending stiffness is underestimated. The calculated value reaches roughly 60% of the experimental results.

REFERENCES


