



Non-Linear modeling of the heel joint of metal plate connected roof trusses

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ABSTRACT

Traditional truss design and analysis assumes pinned joints at the connections. Other design provisions recommend that the trusses be designed and analyzed as frame structures, with careful consideration of the stress, timber behavior, and joint response. A large body of knowledge has been compiled detailing truss behavior as it relates to member properties and responses. However, the modeling of the actual metal plate connections is still the least developed. The semi-rigid behavior of the joints must be accounted for to accurately model the behavior of the joints and the truss. This paper details a procedure used to develop a semi-rigid non-linear heel joint model using the software package SAP 2000. The results of the modeling are very encouraging. A good agreement exists between experimental data gathered by Guinther (1998) and the heel joint model developed.

INTRODUCTION

The basis for the design of metal plate connected wooden trusses in the United States is governed by two trade associations. The associations are the Truss Plate Institute (TPI) and the Wood Truss Council of America (WCTA). TPI represents the manufacturers of the metal plate connectors. TPI is responsible for developing and publishing the design and testing methodology for metal plate connected trusses. Both of these associations are recognized by the building code agencies, and as a result most building codes reference the design procedures set forth by TPI and WCTA.

Truss design by the TPI simplified method relies on empirical data to produce the most economical wood truss. The simplified method resembles traditional truss design that a majority of computer software programs utilize. Under the simplified method, all truss loads, both applied and reaction, must occur at a joint and all joints are pinned. With loads occurring solely at the joints, the truss members only experience axial forces. To account for bending in truss members an additional series of equations was added to approximate these stresses. In traditional truss design the bending stresses are considered to be a secondary stress in most applications.

Traditional truss design and analysis assumes pinned joints at the connections as well. Other design provisions recommend that the trusses be designed and analyzed as frame structures, with careful consideration of the bending stress, timber behavior, and joint response (Aasheim 1991). The analysis of the truss as a frame member forces all joint to be fixed and produces bending stress in the truss members. A large body of knowledge has been compiled detailing truss behavior as it relates to member properties and responses. However, the modeling of the actual metal plate connection is still the least developed. A technique for modeling the semi-rigid non-linear behavior of the heel joint will be discussed in this paper. The semi-rigid behavior of the joints must be accounted for to accurately model the behavior of the joints and the truss.

PREVIOUS RESEARCH

Most of the current research details the attempt to produce an empirical formula for predicting wood truss behavior and the effects of the variable wood properties. Only two facilities, Forest Products Laboratories and Bucknell University, have conducted a series of full-scale tests on heel joints and trusses.

The main difficulty in researching wood trusses arises from the large variability in the properties affecting wood behavior. Previous researchers note the difficulty in modeling and predicting their behavior due to the large amount of variables involved. A list of some of the variables that affect the joint behavior are listed below:

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- Size and number of teeth
- Plate thickness and orientation
- Grain orientation and direction of loading
- Lumber species
- Specific gravity (SPG) of the lumber
- Moisture content (MC) of the lumber
- The joint pressing force used to seat the plate
- The time between fabrication and testing

Several researchers have been able to correlate general trends among the lumber properties. McCarthy and Wolf (1987) noted that the specific gravity (SPG), moisture content (MC) and the modulus of elasticity parallel to the grain (MOE) have significant effects on the parameters needed to define truss joint behavior. King and Wheat (1987) concur with McCarthy's conclusion that SPG is highly correlated to lumber strength and stiffness, especially within lumber species. Poutanen (1988b) notes however, that when the influence of the plate eccentricity is considered, changing the MOE or SPG has little if any effect on the eccentric movement of the plate.

Researchers agree that contact forces occurring at the intersection of members during loading alters the response of the joints. Since the fabrication of the truss joints is not set to standard tolerances, gaps occur between the two lumber members joined by the metal plate connector. Under load the gap tends to close. When the gap closes, contact occurs between the connected members, creating a new path to transfer load. The new load path alters the joint characteristics and the response changes. In general, the presence of the contact transfer point or points tends to increase the strength and stiffness of the joint.

It is also generally agreed upon that the area of plate contact with the wood affects the behavior of the joint. According to Maragechi, K. and Itani R.Y. (1984), the rotational stiffness of the joints is directly proportional to the area of contact between the plate and the connected wood members. Therefore, it can be inferred that in order to create a stiffer joint with regards to rotation, a larger metal plate should be used.

King and Wheat (1988) reported that the bending moments in the members are sensitive to the connection stiffness. Therefore, larger metal plate connectors increase the joint stiffness producing larger member bending moments. King and Wheat also reported that axial forces in truss members are relatively independent of connection stiffness, but are greatly influenced by overall truss geometry.

The basic empirical equation used to predict load-displacement response was developed by R.O. Foschi (1979). The equation, shown below, relates joint performance to plate orientation with respect to grain direction of the lumber and the orientation of the applied load.

$$P = (M_o + M_1 |\Delta|) \left(1 - e^{\frac{-k|\Delta|}{M_o}} \right)$$

The equation reduces the load-displacement response to three parameters. These parameters are:

- M_o : an intercept parameter, based on the intercept of the 'plastic' section of the curve to the load axis.
- M_1 : a stiffness parameter based on the plastic part of the inelastic curve
- k : a stiffness parameter based on the initial or linear stiffness of the curve

Deierlein and others (1990) developed a similar moment-rotation equation for use with flexible steel connections. The equation, shown below, is similar in form and shape to the Foschi model and is easily applied to the rotational response of heel joints.

$$M = \frac{(K_e - K_p)|\Theta|}{\left[1 + \left(\frac{(K_e - K_p)|\Theta|}{M_o}\right)^n\right]^{\frac{1}{n}}} + K_p|\Theta|$$

As with the Foschi equation, the Deierlein equation also has 3 parameters. The parameters are:

- M_o : an intercept parameter, based on the intercept of the ‘plastic’ section of the curve to the load axis.
- K_p : the plastic stiffness of the response curve
- K_e : the elastic stiffness of the response curve
- n : curve shape parameter

The Deierlein equation works well in application on truss heels, however, still suffers the same drawbacks as the Foschi equation. Mainly, it is extremely costly to determine the appropriate parameters through experimentation. The Deierlein equation will be the primary equation for the development of the semi rigid heel joint model in this paper. The Deierlein equation was chosen since it directly relates moment and rotation, as opposed to the Foschi equation that relates load and displacement. Also, the modeling program, SAP 2000, was developed primarily for steel and concrete, so the parameters were easier to match.

PREVIOUS EXPERIMENTAL TESTING

Bucknell University, through funding by MiTek Industries, conducted an extensive truss-testing program. The goal of the research program was to gather experimental data focusing on the stiffness of roof trusses. “The primary objective... was to create a database that would aid in the verification and refinement of methods for truss design and provide further insight into the behavior of wood trusses under loading.” (Guinther 1998). Two concurrent phases of testing were carried out.

The first phase focused on full scale testing of eighteen (18) 30-ft Howe Trusses. The trusses were fabricated using S-Dry, MSR 2400 F_B, 2.0 E, SPF 2x4 dimension lumber. Displacements, loads and strains were continuously measured during testing through the data acquisition computer system.

The second phase of the research consisted of the evaluation of stiffness of several typical truss joints. A series of experiments were performed to evaluate the tension stiffness of the metal plate connector. A total of 75 specimens were tested. The 3x6 metal plates tested were MiTek M-20’s. The metal plates were 20 gage ASTM 466 grade C structural steel. The yield strength and ultimate strength of the plate was 40 ksi and 55 ksi, respectively. The connecting lumber was S-Dry, MSR 2400 F_B, 2.0 E, SPF 2x4 dimension lumber. A second series of experiments were performed on heel joint specimens. With a specially fabricated pull arm, heel joint specimens could be tested in both compression and tension to determine the rotational stiffness of the joints. A total of 48 heel joints were tested. The material properties of the heel joints were identical to the previously stated properties of the plate and the lumber.

SEMI-RIGID HEEL JOINT MODEL

The development of a functional model of a 4/12 Howe truss non-raised heel joint will be discussed in this paper. The structural analysis program SAP 2000 ver. 6.11 nonlinear will be used to model the truss heel. The model should correspond to the results of the experimentation carried out by Guinther at Bucknell. The Deierlein Curve, with parameters determined by Guinther, will be used for the moment-rotation relationship. Of the 48 heel joint tests, eight (8) of which were on 4/12 non-raised heel joints. The data gathered is shown in Table 1.

Included in the table is the specimen identification as labeled from the original tests and the parameters for the best fit Deierlein curve and their corresponding R² values. Based on the ‘poorness’ of fit of specimen D6-COMP (R² of 0.839) and the specimen’s extremely large K_e value, the specimen will not be considered in the modeling process. It is very likely, based upon the large initial stiffness that a large contact force developed in the heel joint that was not present in the all tests.

After the data was examined for possible outliers, such as the D6 specimen, a possible correlation between the tension and the compression specimens was investigated. It was suspected that no significant difference between the tension and compression tests should be present as long as the plate area and contact forces were relatively similar in each specimen. Table 2 illustrates the averages and standard deviations of the tension, compression and all tests.

The difference between the tension and compression heel joint test results is negligible. A further check of the similarity of the data set was tested using a standard hypothesis test procedure for small data sets. To test the null hypothesis of no difference a two sided *t*-test for *Normal Distributions with Unequal Variances* with a level of significance of $\alpha=20\%$ was used. The hypothesis test demonstrated no statistical reason to treat the tension specimens and the compression specimens differently. Therefore, for the purposes of modeling, all of the tension and compression values will be grouped and analyzed as a single unit.

In order to produce a 'standard' heel joint result that could be used as a benchmark for the computer model comparisons, it was necessary to fit the seven Deierlein curves with a best-fit curve. The production of a best-fit curve presented a problem however. As pointed out by McCarthy and Wolf (1987) there is no standard method for fitting curves to the data, and depending on the method used, the parameters may vary. Therefore, an approximate visual 50% bootstrap type method was used. The object was to visually fit a curve that was the average of all the curves; i.e. ran approximately through the center of the data set. After performing the trial and error process several times, a best-fit curve was established. The set of Deierlein curves (the best-fit curve) and the average parameter curve are shown in Figure 1.

The parameters for the best-fit line, as they correspond to Deierlein's equation, are:

- $K_e = 600,000$ in-lb./rad
- $K_p = 30,000$ in-lb./rad
- $M_o = 10,000$ in-lb.
- $n = 2$
- $K_p/K_e = 0.05$

The parameters have a post-yielding slope ratio of 5%. The parameters determined by the best-fit curve will serve as the basis for the modeling of the heel joint.

The structural analysis package SAP 2000 Non-linear version 6.11 was used to create all the computer models. SAP 2000 produces non-linear results for dynamic load conditions and performs the non-linear analysis over a time-history load. In order to mimic the loads applied in the actual experiment at Bucknell, a standard ramp function can be used. The ramp function increases the load, and thus the moment, producing a load condition similar to the experiment.

The second requirement for non-linear modeling is the use of a non-linear link element (Nnlink). These Nnlink elements are the sole part of the model that will show non-linear behavior. SAP possesses six different types of Nnlink elements, each with a specialized use. The element used in modeling the heels was Plastic1. Although originally designed to replicate the non-linear response of steel, the Plastic1 element can be altered to match the form of a Deierlein curve. There are several properties that need to be defined in order for the Nnlink element to function properly, these properties include:

- Rotational inertia about all three axes, for out-of-plane or restrained axes, a value other than zero still needs to be entered.
- Elastic (linear) stiffness
- Plastic (nonlinear) stiffness
- Yield strength
- Post yielding stiffness ratio
- Equation exponent, n

The premise behind constructing a heel model was as follows. The two chords, top and bottom, would be represented by rigid frame elements. The assumption of rigid chords matches the experimental assumption made by Guinther at Bucknell. All of the deformation in the model would be limited to the rotational spring connector at the heel. The spring

connector would be a Plastic1 Nnlink element. Although a single nail/spring does not match the standard method of reporting the values of the parameters on a per tooth basis, it still confirms with the Foschi premise that all deformations are limited to the spring or nail connector.

After a working 4/12 non-raised heel joint model was established, a trial and error process was used to determine the parameters that best fit the data. Again, a visual method was used to determine the quality of the fitted line. Each SAP run was plotted against the experimental curve until a 'good' fit was obtained.

Each successive SAP trial brought the model closer to the experimental data. The final parameters of the SAP model to produced the results are shown in Table 3.

The above parameter values represent the best-fit line generated by the SAP 2000 model. These parameters will be used to model the heel in the full-scale truss models. It should be noted that Table 4 contains only the necessary values to define the model. Other values can be entered in the Nnlink element plastic1, but they are excluded from the discussion because they are not necessary to define the model. Examples of such values are, the wood density, rotation inertia about axis 1 (a non-essential excluded axis), self weight, etc.

As was demonstrated in the previous section, the results of the heel joint model compare favorably to the experimental results. The results of the heel joint model can be seen in Figure 2. The graph compares the actual experimental data fitted to the Deierlein curve to the model data generated by SAP 2000. The visual comparison of the two curves is very good, with both overlapping each other.

In order to confirm the quality of the curve fit, a percent difference method was used. The absolute percent difference between the Deierlein experimental curve and the SAP model curve was used. A level of 10% error was set as the guideline for an acceptable fit of the data. The graph of the percent error versus the degree of rotation of the heel joint is depicted in Figure 3.

As can be seen in the preceding graph, the percent error falls below the acceptable level of 10% for the entire curve. The percent error is greatest in the linear range (rotation less than 0.01 radians) and drops to a value of less than 1% in the non-linear range (rotation greater than 0.01 radians). The percent error decreases along the length of the curve, therefore the predicted values in the non-linear range are better than the ones in the linear range. The humps in the curve occur because the error oscillates about the predicted value and appears to converge for rotation less than 0.04 radians. Based on the visual fit criteria and the percent error, the heel joint model in SAP 2000 is an effective and accurate representation of the experimental results.

CONCLUSIONS

There are several conclusions that can be drawn from the performance of the heel joint model. The computer model generated by the use of SAP 2000 Non-linear version 6.11 proved to be an effective and accurate method of modeling non-linear heel joint behavior. The properties used in the heel joint Nnlink element correspond well with the properties reported by Guinther.

Table 4 lists the properties for both the Deierlein equation and their SAP 2000 counterpart. In general the properties used in the Deierlein equation compare well to the Nnlink properties required by SAP 2000. The differences in the values for the parameters are negligible, especially when the large scatter in the experimental data is considered. The only problem arises on the determination of the proper rotational inertia to be used in the SAP 2000 model. No valid counterpart could be determined in the experimental study or Deierlein's model that corresponds to the rotation inertia of 1000 in^4 used by the SAP model. Further investigation is required to determine a suitable method, other than trial and error, to determine this value.

It is reasonable to conclude, based on the good comparison between the SAP 2000 model and Deierlein's predicted response from the Bucknell data, that SAP 2000 is an effective method of modeling the non-linear behavior of truss heel joints. Further, reducing the non-linear behavior to a single element simplifies the modeling process and can be implemented into full-scale trusses currently in use today. It is likely that the application of the Nnlink element can be expanded to include other types of joint behavior and response, not just rotational stiffness and deformation.

Non-linear analysis is recommended for trusses of longer clear spans and larger loads. As the truss span increases the fixed heel joint moment increases to values that will produce significant non-linear effects. These non-linear effects will have an impact on both deflections and stresses.

RECOMMENDATIONS FOR FURTHER STUDY

A myriad of opportunities for continued study presents themselves in the area of metal plate connected wood truss design. Further investigation is required to determine the precise relationship between the variables, such as SPG, MC, etc., which affect the strength and stiffness of the joint. The position of the plate and its relative effects on joint performance needs to be quantified. The stresses in the truss as a result of the degree of rigidity of the joint should be investigated. Likewise, further study on the effects of span length is needed. Better and more in depth non-linear models should be developed to better predict the truss behavior.

As can be readily seen, the field of metal plate truss design is still relatively young, thus a large amount of research opportunities are present for the ambitious researcher. There is a current need in the field to better understand the non-linear behavior of the metal plate connected truss. Once a better understanding is reached, more efficient truss designs in the area of wood strength and plate size and placement can be fabricated.

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TABLES

Table 1: Non-Raised Heel Joint Data

Specimen	K _e	K _e per Tooth	K _p	K _p per Tooth	M _o	M _o per Tooth	n	R ²	
	lb-in/rad	lb-in/rad/tooth	lb-in/rad	lb-in/rad/tooth	lb-in	lb-in/tooth		Value	
D2-TEN		616653	21209	102916	453	5151	23	2.22	0.993
D5-TEN		887722	31507	-98724	-451	19269	88	1.00	0.998
D7-TEN		1064362	37268	6907	30	10889	48	2.35	0.989
D3-COMP		392457	13583	-1024491	-4360	107463	457	2.97	0.996
D4-COMP		332680	11724	-414143	-1857	48335	217	3.35	0.999
D8-COMP		1533516	53221	336902	1440	1581	7	1.00	0.999
D6-COMP		3186054	108988	2111081	898	6756	29	1.00	0.839
D1-COMP		451983	15612	-43079	-186	14011	61	1.00	0.991

Note: TEN denotes tension specimens and COMP denotes compression specimens

Table 2: Averages and Standard Deviations of 4/12 Non-Raised Heel Test Specimens

Specimen	K _e	K _e per Tooth	K _p	K _p per Tooth	M _o	M _o per Tooth	n	R ²	
	lb-in/rad	lb-in/rad/tooth	lb-in/rad	lb-in/rad/tooth	lb-in	lb-in/tooth		Value	
<i>Tension Specimen Averages and Standard Deviations</i>									
Average	856246	29995	3700	11	11770	53	1.857	0.993	
Std. Dev.	225508	8136	100858	452	7100	33	0.745	0.005	
<i>Compression Specimen Averages and Standard Deviations</i>									
Average	677659	23535	-286203	-1241	42848	186	2.080	0.996	
Std. Dev.	572646	19854	579887	2477	47398	202	1.257	0.004	
<i>All Specimen Averages and Standard Deviations</i>									
Average	754196	26303	-161959	-704	29528	129	1.984	0.995	
Std. Dev.	435919	15201	442196	1893	37630	160	0.994	0.004	

Table 3: Parameters of the Final 4/12 Heel Joint Model Created in SAP 2000

Rotational Inertia	Linear Stiffness, K _e	Non-Linear Stiffness, K _p	Yield Strength	Post-Yielding Ratio	Curve Exponent
in ⁴	kip-in/rad	kip-in/rad	kip-in		n
1000	650	181	10.5	0.0275	1.25

Table 4: Comparison of the Reported Experimental Parameters and the Heel Joint Model Parameters

	Linear Stiffness, K _e	Non-Linear Stiffness, K _p	Yield Strength	Post-Yielding Ratio	Curve Exponent
	kip-in/rad	kip-in/rad	kip-in		n
Deierlein Best Fit Curve	600	30	10	0.05	2
SAP Model	650	181	10.5	0.0275	1.25

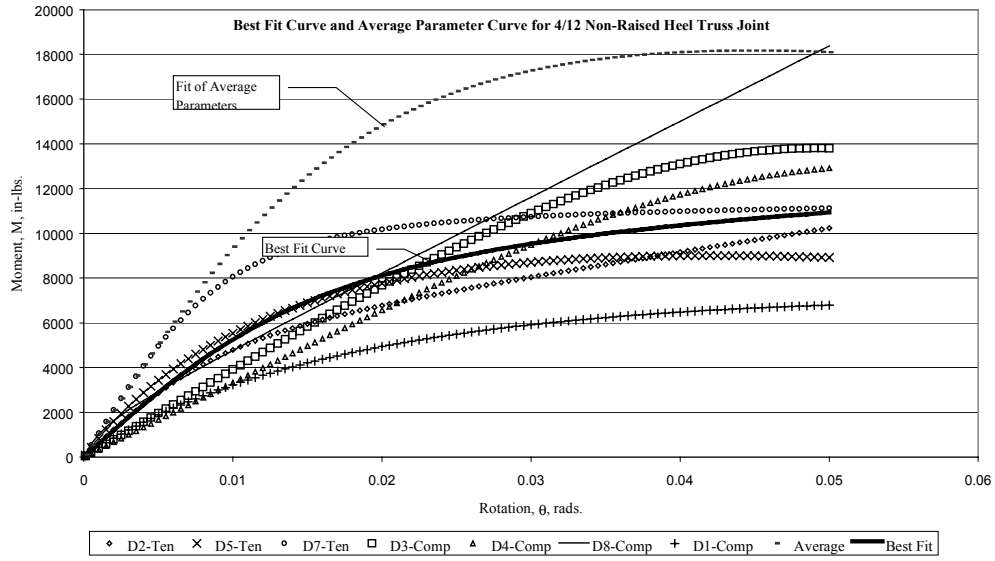


Figure 1: Graph of all the experimental Deierlein curves generated by Guinther, as well as the best-fit curve.

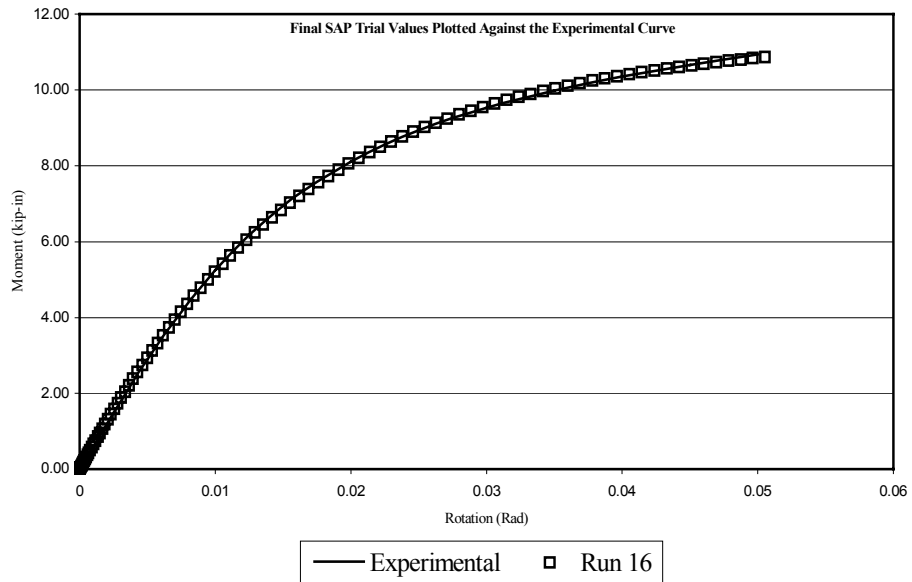


Figure 2: Graph of the curve generated by the SAP 2000 model against the Best fit Deirlein curve.

Percent Error Between the SAP Model Values and the Predicted Experimental Values

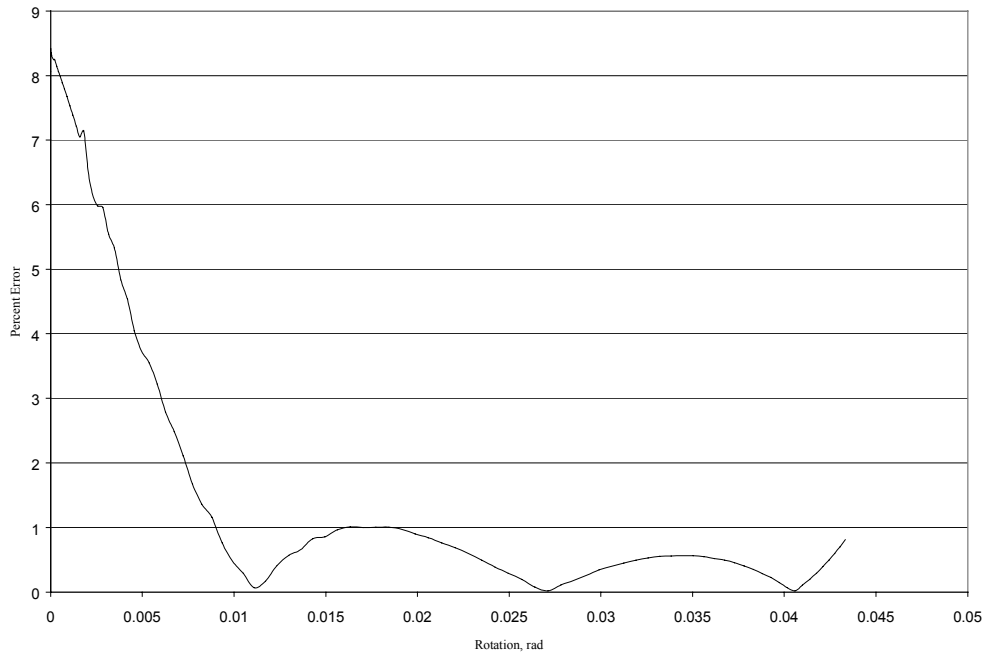


Figure 3: Graph of the absolute percent error as the rotation angle increases.